HYPER VERSION

Received: *November 14, 2001*, Accepted: *November 22, 2001*
Revised: *November 22, 2001*

Inflation and the dS/CFT correspondence

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Abstract: It is speculated that the observed universe has a dual representation as renormalization group flow between two conformal fixed points of a three-dimensional euclidean field theory. The infrared fixed point corresponds to the inflationary phase in the far past. The ultraviolet fixed point corresponds to a de Sitter phase dominated by the cosmological constant indicated in recent astronomical data. The monotonic decrease of the Hubble parameter corresponds to the irreversibility of renormalization group flow.

Keywords: AdS-CFT Correspondance, Cosmology of Theories beyond the SM.

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Recent observations [1]–[3], together with the theory of inflation, suggest the possibility that our universe approaches de Sitter geometries in both the far past and the far future, but with values of the cosmological constant that differ by a hundred or so orders of magnitude. In recent theoretical work [4], it was conjectured that a fully quantum theory, including gravity, in pure de Sitter space with a fixed cosmological constant has a certain dual representation as a conformally invariant euclidean field theory on the boundary of de Sitter space.[1] The purpose of this short note is to extend this "dS/CFT correspondence" so as to potentially include our own universe.

We begin with a brief synopsis of the relevant portions of [4] for the case of four dimensional de Sitter space, dS_4.[2] We assume that all observables can be generated from the complete set of quantum correlation functions whose arguments lie on the asymptotic boundary of dS_4. For a cosmology like our own the relevant boundary is $\hat{\mathcal{I}}^+$, the euclidean $\mathbb{R}^3$ at future infinity.[3] The correlators on $\hat{\mathcal{I}}^+$ of course must transform covariantly under general coordinate transformations. Of particular interest are the $SO(4,1)$ coordinate transformations which are isometries of dS_4. It turns out that these act on $\hat{\mathcal{I}}^+$ as the $SO(4,1)$ conformal group of euclidean $\mathbb{R}^3$. The conjecture is that the $\hat{\mathcal{I}}^+$ correlators are generated by a three-dimensional conformal field theory whose conformal group is identified with the dS_4 isometry group. That is, there are two dual representations of the same theory, one as a bulk quantum theory of gravity on dS_4, and one as a purely spatial conformal field theory without gravity on $\mathbb{R}^3$.

The conjecture was motivated in [4] by an analysis of the so-called asymptotic symmetry group of de Sitter space and its generators, together with an appropriately crafted analogy to the by now well-established AdS/CFT correspondence [18]–[21]. A primary difficulty with the dS/CFT conjecture is the absence of any well-controlled example of a de Sitter solution to string theory (or other form of quantum gravity) in which concrete calculations are possible and the conjecture can be tested. There is no direct construction of the boundary field theory, although some general characteristics can be indirectly deduced such as the relation between conformal weights and particle masses and the absence of unitarity. We hope this situation will be remedied in the near future perhaps by the construction of appropriate de Sitter solutions to string theory. Meanwhile, the proposal resonates well with other ideas from string theory and seems worth pursuing, which we shall now proceed to do without further apology.

[1]This conjecture followed related observations in [5]–[17], and was inspired by an analogy to related early [18] and modern [19]–[21] results obtained in AdS (anti-de Sitter space). See also [22, 23].

[2]Reference [4] focused on the case of three spacetime dimensions because of the extra control provided by the infinite-dimensional enhancement of the conformal group for that case.

[3]For a discussion of other boundaries see [4], and footnote 4 below.

An intriguing feature of the dS/CFT correspondence is the identification of time evolution in the bulk with scale transformations in the boundary. In flat Robertson-Walker coordinates the dS_4 metric is

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2 \,, \tag{1}$$

where H is the Hubble constant. This geometry is isometric under the modified time translations

$$t \to t + \lambda \,, \qquad \vec{x} \to e^{-\lambda H} \vec{x} \,, \tag{2}$$

whose generator we will refer to as $\mathcal{H}$. From the first term in (2) we see that $\mathcal{H}$ generates time evolution in the bulk gravity theory, while from the second term we see that it generates scale transformations in the boundary theory. Furthermore late times in the bulk correspond to the UV (ultraviolet) of the boundary field theory, while early times correspond to the IR (infrared).

Let us now assume that our universe is well-approximated by a geometry of the Robertson-Walker form

$$ds^2 = -dt^2 + R^2(t) d\vec{x}^2 \,, \tag{3}$$

where at early times

$$t \to -\infty \,, \qquad \frac{\dot{R}}{R} \to H_i \,, \tag{4}$$

while at late times

$$t \to \infty \,, \qquad \frac{\dot{R}}{R} \to H_f \,. \tag{5}$$

H_i here is the inflationary era Hubble constant, typically taken to be of order $10^{24}\,\mathrm{cm}^{-1}$. H_f is the final value of the Hubble constant which recent observations indicate may be of order $10^{-28}\,\mathrm{cm}^{-1}$. At intermediate times $R(t)$ corresponds to standard big bang cosmology.

For such a function $R(t)$, the universe has no isometry of the form (2), and there would be no reason to expect a dual representation of the bulk gravity theory as a boundary conformal field theory. In order to interpret this, we again take our cue from parallel developments in the study of AdS [24]–[26]. The absence of a bulk isometry is conjectured to correspond to a boundary field theory which is not conformally invariant. Bulk time evolution is dual to RG (renormalization group) flow in the boundary field theory. Since the isometry (2) is recovered for $t \to \pm\infty$, the RG flow begins at a UV (ultraviolet) conformally invariant fixed point and ends at an IR (infrared) conformally invariant fixed point. We note that since late (early) times corresponds to the UV (IR) RG flow corresponds to evolution back in time from the future to the past.[4]

[4]One might have expected a discussion based on the global picture of de Sitter space as a contracting/expanding sphere, and the associated global time which runs from $-\infty$ on $\mathcal{I}^-$ to ∞ on

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To summarize so far, it is conjectured that our universe is an RG flow between two conformal fixed points. Time evolution is inverse RG flow.

The Hubble parameter

$$H(t) = \frac{\dot{R}}{R} \tag{6}$$

plays a special role in the boundary theory. To understand this we first consider the case of dS_3. It was shown in [4] (using properties of the Virasoro algebra and an analysis of the asymptotic symmetry generators) that the central charge of the two-dimensional boundary theory obeys

$$c_2 = \frac{3}{2HG_3}\,, \tag{7}$$

where G_3 is the three-dimensional bulk Newton constant. c_2 is a measure of a number of degrees of freedom of the theory and is related to the two point function of the stress tensor. For a generic unitary (non-conformal) two-dimensional field theory a "c-theorem" has been proven by Zamolodchikov: an appropriately defined c_2 always decreases from the UV to the IR along RG flows [27].[5] Intuitively this corresponds to the fact that degrees of freedom are integrated out along RG flows and hence c should decrease. Indeed an appropriate combination of Einstein's equations can be written in the form

$$\dot{H} = -\frac{8\pi}{3}G_3(p+\rho)\,. \tag{8}$$

If the null energy condition is obeyed, the right-hand side is non-positive and H decreases with time. c_2, as defined by (7) with a general time-dependent H, will then decrease along RG flows (which go from the future to the past).

The connection between the c-theorem of [27] and (8), although persuasive in the AdS/CFT context, is here obscured by the fact that the field theories arising in the dS/CFT context are not likely to be unitary [4]. Unitarity was crucial in the proof of [27]. Nevertheless, we would like to interpret c_2 as a measure of the number of degrees of freedom of the boundary theory, which does decrease along RG flows regardless of whether or not the theory is unitary.

A similar discussion, parallel to that given for AdS_d [24]–[26],[6] can be made for higher d-dimensional de Sitter spaces and yields $c_{d-1} \sim 1/H^{d-2}G_d$. The Einstein equation then implies that this decreases along RG flow, interpreted as inverse time

$\mathcal{I}^+$. There are several problems with this. The first is that this picture is time reversal invariant and hence at odds with the irreversibility of RG flow. Secondly, and more importantly, evolution along this global time is not part of any of the de Sitter isometries. Hence there is no way to associate it to a conformal transformation of the boundary field theory.

[5]The c_2 defined in [27] is not exactly the same as that defined by (7), but they agree at the conformally-invariant fixed points.

[6]In higher dimensions, despite numerous attempts, there is no general proof from field theory that an appropriately defined c_{d-1} decreases along RG flow. The results from AdS/CFT support the idea that such a c-function should exist.

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evolution. For dS_4 an appropriate combination of Einsteins equations can be written in the form

$$\dot{H} = -4\pi G(p + \rho) . \tag{9}$$

If the null energy condition is obeyed, the right hand side is non-positive, H decreases with time and c_3 decreases along RG flows.

One of the puzzling features of an expanding universe is that the number of degrees of freedom seems to increase with time. If string theory or an alternative provide a cutoff at the Planck length, then the number of degrees of freedom of the universe at any moment of time is naively of order the spatial volume in Planck units.[7] This increase in the number of degrees of freedom seems hard to reconcile with the existence of a unitary Hamiltonian. In the present proposal, the extra degrees of freedom arise because time evolution to the future is inverse RG flow, and hence corresponds to integrating in new degrees of freedom.

A fundamental puzzle in modern cosmology is the large hierarchy characterized by the large ratio $H_i/H_f \sim 10^{52}$. In the present proposal this turns into the field theory problem of understanding the large ratios of the numbers of degrees of freedom of the UV and IR fixed points. This is in a sense the opposite of the recent discoveries of [28, 29] that field theory hierarchies could be turned into problems in geometry.

The picture proposed here recasts the question "What is the origin of the universe?" in a new light. In a sense it puts the origin of the universe in the infinite future, rather than the infinite past, because only in the ultraviolet can all the degrees of freedom comprising the universe be seen. The past is an IR fixed point at which most of the degrees of freedom are not present. Indeed one could imagine a preinflationary stage with no degrees of freedom at all, corresponding to a dual field theory at $\mathcal{I}^+$ with a mass gap and trivial IR fixed point. In our picture the universe is being continually created with the passage of time, and come into full existence only in the infinite future.

In conclusion, recent indications that our universe is asymptotically dS_4 in both the past and future are tailor made for a new cosmological paradigm of the universe as RG flow.

Acknowledgments

It is a pleasure to thank Nima Arkani-Hamed, R. Bousso and Shiraz Minwalla for useful discussions. This work was supported in part by DOE grant DE-FG02-91ER40655.

[7]The generalized second law/holographic principle may highly constrain the possibilities for exciting many of these degrees of freedom at the same time.

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